Vol. 14, 2025, 95-105

DOI: 10.17512/bozpe.2025.14.10

Construction of optimized energy potential
Budownictwo o zoptymalizowanym potencjale energetycznym

ISSN 2299-8535 e-ISSN 2544-963X

Weighted average as a basis for determining the reliable tilt of a multi-segment building undergoing rectification

Krzysztof Gromysz[1*] (*orcid id: 0000-0002-7778-8620*)
Marta Kadela[2] (*orcid id: 0000-0003-2127-0061*)
Marian Drusa[3] (*orcid id: 0000-0003-3372-6441*)
Štefan Kovalčik[3] (*orcid id: 0009-0000-2988-4821*)
[1] Silesian University of Technology, Poland
[2] Building Research Institute (ITB), Poland
[3] University of Žilina, Slovak Republic

Abstract: Mining exploitation of hard coal occurs in the Silesian region of Poland, resulting in the formation of post-mining voids that cause uneven lowering of the ground level. Consequently, thousands of buildings are tilted from vertical. The tilt of a structure is most often considered a phenomenon that changes the technical properties of buildings, reducing their comfort of use and decreasing the value of the real estate. Tilting induces structural damage in wall-bearing systems. Furthermore, this tilt complicates the use of buildings, reduces their value, and in extreme situations leads to exceeded limit states. Therefore, such buildings are rectified through non-uniform elevating by means of hydraulic piston jacks built into the walls of buildings. Based on the analyzed rectification processes, a method for determining the reliable tilt of a multi-segment building has been proposed. It is important to note that the tilt of individual elements within the segments, as well as the tilt of the segments themselves, can differ significantly. This analysis demonstrates that the reliable tilt of a multi-segment building should be calculated as a weighted average. The values of the weights can result from the plan area of the segments, the volume of the segments, or their significance in terms of the functioning of the building. In the case of the five-segment school building analyzed in this article, three reliable tilts were determined due to the position of the expansion joints and the functional solutions. One reliable tilt as a weighted average was determined for the three segments. The rectification of the other two segments was carried out separately, based on the reliable tilt deter-mined independently for these segments.

Keywords: vertical deflection of a building, rectification, reliable tilt of a multi-segment building, weighted average

Please, quote this article as follows:

Gromysz, K., Kadela, M., Drusa, M. & Kovalčik Š. (2025) Weighted average as a basis for determining the reliable tilt of a multi-segment building undergoing rectification. *Construction of Optimized Energy Potential* (*CoOEP*), 14, 95-105. DOI: 10.17512/bozpe.2025.14.10

---

[*] Corresponding author: krzysztof.gromysz@polsl.pl

Introduction

The tilt of a building from vertical is a common phenomenon observed in every region of the world. It affects various types of structures, including residential buildings (Kijanka & Kowalska, 2017), industrial structures (Dudek et al., 2021), historic masonry towers (Macchi, 2005), timber towers (Gromysz, 2019a) and timber/brick towers (Zhang et al., 2018), chimneys (Kaszowska et al., 2018), churches (Gromysz, 2019b), reservoirs (Gromysz, 2021), transmission towers (Liu & Liu, 2008), grain elevators (Baracous, 1957), and even high-rise buildings (Maffei & Gonçalves, 2016), bridge supports (Shi et al., 2013), piles (Peng, 2017), as well as high-voltage poles (Yuan et al., 2012) and even oil rigs (Rapoport & Alford, 1989) that are currently under construction.

The most common cause of tilting building structures from vertical is uneven subsidence of the ground. Notable examples include the 55 m high Leaning Tower of Pisa (Geng et al., 2020; Terracina, 1962), the 56 m high church tower in Bad Frankenhausen (Jänichen et al., 2020), the mid-14th century Oude Kerk church tower in Delft (Dabrowski et al., 2023), and the 500-year-old wooden tower of St. Bavo Church in Haarlem, Netherlands (Truong-Hong et al., 2021), which stands at 45 meters and has an approximate deflection of 0.63 meters. In Poland, the most common cause of tilting is underground coal mining (Lamich et al., 2016). As a result of this exploitation, uneven lowering of the ground surface of a continuous (Strzałkowski, 2022) and discontinuous (Paszek, 2024) nature occur, resulting in the leaning of structures from the vertical (Orwat, 2020).

The tilt of a structure is most often considered as a phenomenon that changes the technical properties of buildings, including their dynamic properties (Ahmari et al., 2015; Bao et al., 2021), reducing their comfort of use (Kijanka & Kowalska, 2017) and reducing the value of the real estate (Kowal, 2014). Tilting induces structural damage in wall-bearing systems (Strzałkowski, 2019), frame structures (Ren & Yan, 2015), and results in increased structural stress. Furthermore, as demonstrated by (Al' Malul & Gadzhuntsev, 2018), tilting significantly elevates the probability of structural damage and failures.

Instances of tilt measurements, employing traditional geodetic methods, are documented in the works of (Halicka & Zyga, 2019) and (Orwat & Gromysz, 2021). These measurements facilitate the assessment of variations in the tilts and inclinations of individual structural components and enable an evaluation of the precision of building assembly. Ground-level laser measurement techniques permit real-time monitoring of tilt variations, including those induced by thermal effects and wind loads, and allow for the determination of the dynamic characteristics of structures. Conversely, aerial laser scanning provides the capacity to assess roof slopes, thereby enabling the calculation of building tilt (Yonglin et al., 2010).

A considerable number of structures exhibiting tilt are stabilized through foundation reinforcement techniques, such as the installation of driven micro-piles (Elsawwaf et al., 2023), steel piles (Yin et al., 2011) or reinforced concrete piles, in addition to ground reinforcement strategies (Macchi, 2005). In special situations, when the tilt exceeds 15 mm/m, the tilt is removed by means of hydraulic piston jacks built into the walls of buildings (rectification) (Gromysz, 2017). Rectification involves extending the pistons in the jacks, which results in the building being raised unevenly (Fig. 1a). The elevation of each corner (u_{obj}) depends on the building's tilt (Fig. 1b). However, the tilt is not uniform; the slope of the floors often differs from the tilt of the walls, and each corner may tilt differently. Therefore, before any rectification can occur, it is essential to determine the reliable tilt of the building. This assessment forms the basis for calculating the required elevation adjustments for each corner.

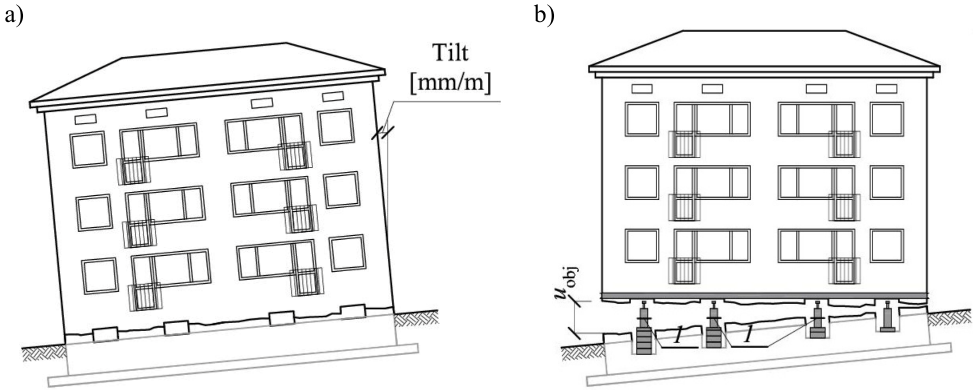

Fig. 1. Object rectification: a) building tilted from vertical, b) building after rectification; *1* – hydraulic piston jack, u_{obj} – high of the corner elevation (*own research*)

Rectification through uneven lifting is based on the rotation of an object about an axis that intersects the 'zero' point. In the context of building projections that approximate rectangular shapes (specifically, convex configurations), this 'zero' point corresponds to the highest positioned corner of the structure. The rotation of the building is induced by the differential elevation of its corners. The specific values required for corner elevation (u_{obj}) are derived from the assessed reliable tilt of the structure (Fig. 2).

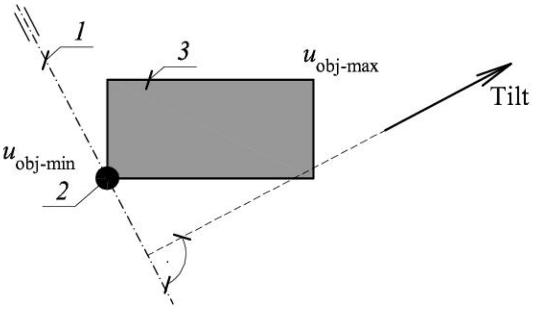

Fig. 2. Principle for determining corner elevation in a convex plan building:
1 – axis of rotation of the building, *2* – "zero" point of the segment,
u_{obj} – high of the corner elevation, *3* – rectified building (*own research*)

For single-segment buildings with a load-bearing wall system, the method for determining the reliable tilt has already been developed (Gromysz, 2023). In single-storey buildings in particular, the wall tilt is the reliable value. In single-storey buildings with a usable attic, the slope of the attic floor is the reliable value. When planning the non-uniform elevating height of multi-storey buildings, the tilt from the vertical of the lift shafts should be taken as a reliable value. If there are no lifts in a multi-storey building, the tilt of an undeformed structural element can be used as a reliable reference. For buildings with rigid floor structures, the change in floor slope equals the change in wall tilt. In multi-segment buildings, there is no single method for determining the reliable deflection. This paper presents the case of a multi-segment building where the individual segments are functionally connected. The objective function for determining the reliable tilt of the building aimed to maintain its functionality after rectification.

1. Description of the building and the tilt of its segments

The subject of this analysis was a school building comprising of five segments labeled “A”, “B”, “C”, “D”, and “E” (Fig. 3). The building exhibited a T-shaped plan with a total area of 1.281 m^2, which can be inscribed within a rectangle measuring 79.8 m by 37.7 m. The individual areas of the segments varied, with segment “B” encompassing 63 m^2 and segment “A” covering 354 m^2 “A” (Table 1). Segments “A”, “B”, and “C” contained classrooms that were accessible via corridors. Notably, these corridors traversed the expansion joints located between segments “A” and “B”, “B” and “C”, as well as “A” and “C”. The floors in these segments were on one level, which was taken as ±0.00. Segment “D” accommodated gymnasium facilities and technical rooms, while segment “E” functioned as the gymnasium itself. The floor levels of segments “D” and “E” were situated at –1.22 m. Access from segment “C” to segment “D” was facilitated by seven steps (Fig. 3).

Segments “A”, “B”, and “C” were constructed with a full basement and two above-ground stories. The basement walls of these segments were composed of reinforced concrete, monolithic in design, and were capped with a reinforced concrete ring beam that extended beyond the outer edges of the walls. The sandwich walls of the above-ground stories rested upon this extended ring beam. The columns within these segments were supported by the reinforced concrete basement walls. The roof structure was fabricated from steel rolled profiles, with portions of the segments covered by layers of bituminous paper and others by sheet metal.

Segments “D” and “E” were designed as single-storey structures, featuring an enclosed space beneath. Both segments were supported by reinforced concrete strip foundations, which were connected monolithically to the reinforced concrete foundation walls. Additionally, within these segments, the foundation walls were capped with an extended reinforced concrete ring beam, wherein, specifically for the gymnasium segment (segment “E”), the columns and cores were effectively fixed.

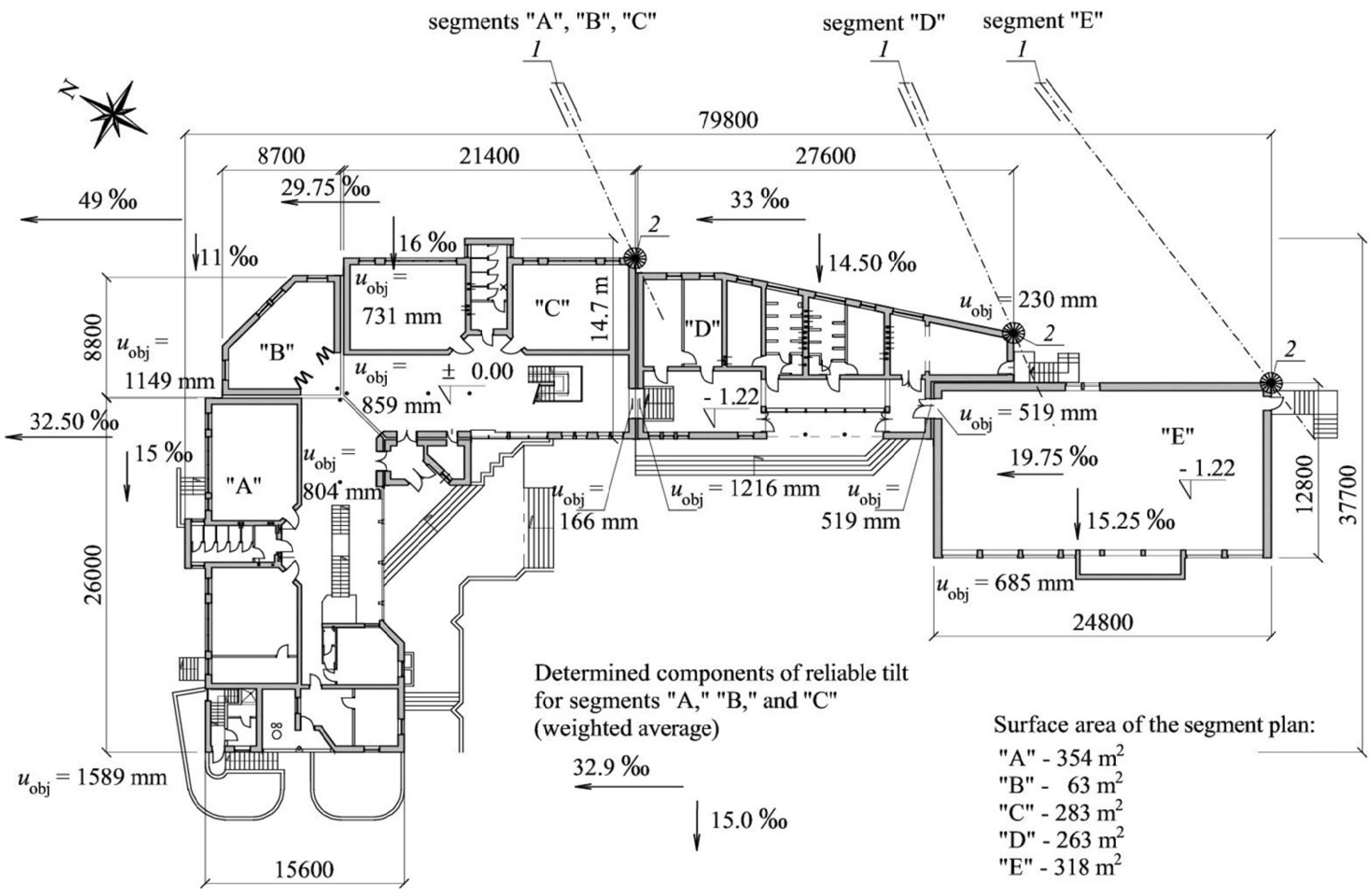

Fig. 3. Plan of the five-segment school building, which features functionally interconnected segments, with the specified rectification parameters and lifting heights (u_{obj}) of the corners; *1* – axis of rotation of the segment, *2* – "zero" point of the segment (*own research*)

The building exhibited a general tilt in a north-western direction; however, this tilt differed among the segments due to varying degrees of mining-induced deformation. This happens particularly in buildings with large projections. The tilt of each segment was evaluated through geometric levelling of the ground floor. Segment "B" demonstrated the greatest tilt towards its longer side at 49 ‰, while segment "E" exhibited the least tilt at 19.8 ‰. In terms of the shorter edge, segment "C" displayed the most significant tilt at 16.0 ‰, whereas segment "B" presented the least tilt at 11.0 ‰. The tilt measurements for each segment are depicted in Figure 3 and summarized in Table 1.

Table 1. Data on the segments of the school building (*own research*)

Segment	Surface area of the segment's plan	Tilt components of the segments	Weight of the segment's tilt	Components of reliable tilt to be removed	The elevation height of the segments corners u_{obj} (min – max)
	m^2	mm/m	–	mm/m	mm
"A"	354	32.5 15	0.506	32.9 15.0	804-1589
"B"	63	49.0 11.0	0.090		731-1149
"C"	283	29.8 16.0	0.404		0-859
"D"	263	33.0 14.5	–	33.0 14.5	230-1216
"E"	318	19.8 15.3	–	19.8 15.3	0-685

2. Determining the reliable tilt of the segments

As mentioned in the introduction, for rigid buildings characterized by a load-bearing wall system, the mean tilt can be regarded as the reliable tilt. For such structures, this tilt can be ascertained by calculating the height differential between the highest and lowest points within a given storey. In the case of the school building under examination, this entailed selecting a single zero point at the southeast corner of segment "E" and evaluating the elevations for the corners of all segments. Implementing this methodology would have resulted in the segment exhibiting the greatest tilt remaining further displaced from the vertical, while the segment with the least tilt would have been oriented in the opposite direction. Conversely, establishing individual zero points for each segment and rectifying each segment independently was inadvisable, as it would have eliminated the tilt of all segments but would have introduced steps within the corridors.

Consequently, the following assumptions were established to determine the reliable tilt. It was decided that the rectification of the building must not result in the emergence of steps between segments. Specifically, the presence of steps in the corridors of the educational segments – namely, between segments “A” and “B”, “A” and “C”, and “B” and “C” – was prohibited. Thus, a uniform reliable tilt was calculated for segments “A”, “B”, and “C”. Conversely, the rectification of segments “D” and “E” could proceed independently, provided that no steps were introduced in the passage between these segments. Variations in the number of steps between segments “D” and “E” was permissible.

To determine a reliable tilt for segments “A”, “B”, and “C”, we first calculated the average slope of these segments. However, this approach proved to be wrong. The significant tilt of segment “B” with its relatively small projection (with a tilt of 49 ‰ and a projection area of 63 m^2), would have disproportionately influenced the corner elevations of segment “A”, which, although exhibiting a smaller tilt, possessed the largest projection area (with a tilt of 32.5 ‰ and a projection area of 354 m^2). It was therefore proposed that the tilt of segments “A”, “B”, and “C” be represented as a weighted average of the floor tilts, using the projected area of each segment as the weighting factor. The values corresponding to these weights are detailed in Table 1. Moreover, it would have been inappropriate to consider any alternative tilt, such as that derived from the edges of the walls, since the objective was to achieve level floors within the segments.

For the single-storey segments “D” and “E”, the tilt of the ground floor was similarly regarded as the reliable tilt. This determination is predicated on the presence of a void beneath the floors, which facilitated the rectification of the building, including the floor itself. Furthermore, achieving alignment of the gymnasium with the ground floor level was prioritized over minimizing the tilt of the columns.

The adoption of independent “zero” points for segments “D” and “E” resulted in a level discrepancy of 230 mm in the passage between these segments. Consequently, it was planned to uniformly elevate segment “E” by this level difference.

Based on the aforementioned considerations, the elevation heights of the corners of the individual segments were calculated. The heights (u_{obj}) of the characteristic points are illustrated in Figure 3. Segment “A” exhibits the highest value of u_{obj} at 1589 mm, while segment “E” has the lowest maximum value of u_{obj} at 685 mm. The maximum and minimum values of u_{obj} corresponding to each segment are summarized in Table 1.

The analyses conducted provided a foundation for the effective rectification of the school building. Notably, the mutual displacements among the segments resulted in a reduction of the height difference between the ground floor levels of segments “C” and “D” from 1220 mm (7 steps) to 174 mm (an imperceptible ramp). This is depicted in Figure 4a and 4b. The rectification undertaken (Fig. 5a) successfully restored the full functionality of the school building (Fig. 5b).

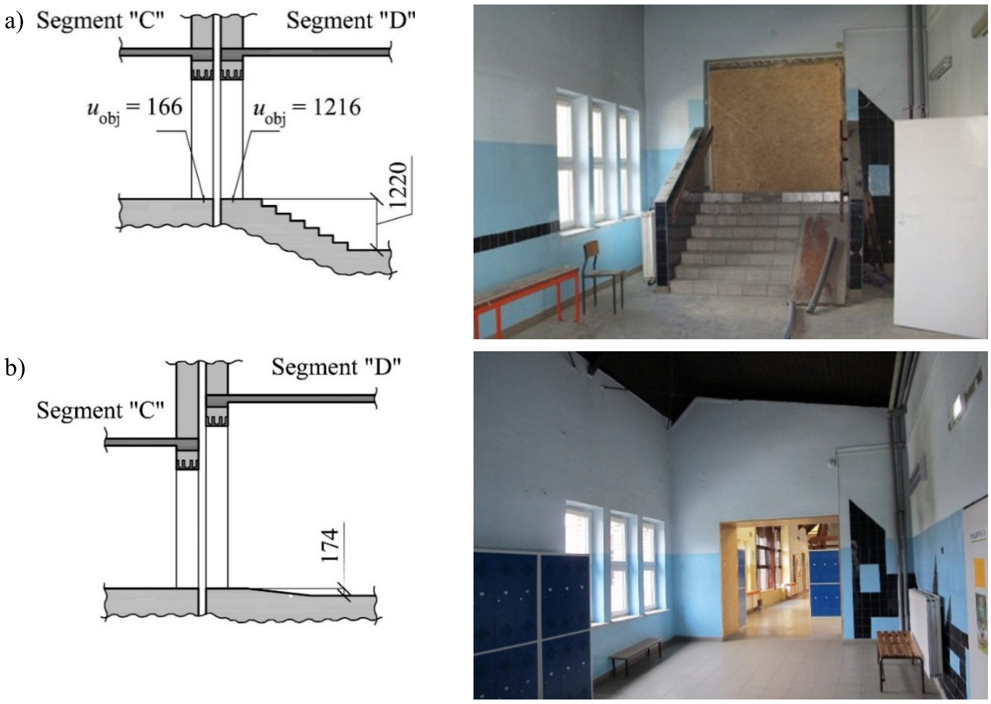

Fig. 4. Mutual positioning of segments “C” and “D” of the school building: a) prior to rectification, b) subsequent to rectification (*own research*)

Fig. 5. School building: a) view from the east, b) object during rectification – jacks built into the walls (*own research*)

Conclusions

The foundation of planning the rectification of tilted buildings is a reliable tilt value. For single-segment structures, this tilt is represented by the average tilt of the walls, the average tilt of the floor of each storey, or the tilt of the lift shafts, contingent upon the building's height and number of floors.

In the context of multi-segment buildings with extensive floor plans, the tilt of individual segments may differ significantly. Research indicates that in such cases, the average tilt of the segments cannot be deemed a reliable indication of the overall tilt of the building. Consequently, the rectification of each segment should be conducted independently. However, this procedure must not compromise the building's functionality; specifically, it is imperative that no additional steps are introduced in the expansion joints. Therefore, under certain circumstances, it may be necessary to establish a singular reliable tilt that encompasses multiple segments. Evidence suggests that this tilt may be represented as a weighted average. The weights can be derived from factors such as the projected area of the segments, their volume, or their operational significance within the facility.

In the analysis of a five-segment school building, three reliable tilts were identified based on the positioning of the expansion joints and functional considerations. A single reliable tilt, calculated as a weighted average, was determined for three of the segments, while the rectification of the remaining two segments was executed independently, based on the reliable tilt established for those specific segments.

Acknowledgements

The publication of this article was supported by: (1) the Rector of the Silesian University of Technology under grant number 03/020/SDU/10-21-01, (2) own funds of Department of Building Structures.

Bibliography

Ahmari, S., Yang, M. & Zhong, H. (2015) Dynamic interaction between vehicle and bridge deck subjected to support settlement. *Engineering Structures*, 84, 172-183. DOI: 10.1016/j.engstruct.2014.11.018.

Al' Malul, R. & Gadzhuntsev, M. (2018) The reliability of multistory buildings with the effect of non-uniform settlements of foundation. *E3S Web Conf.*, 33, 02040. DOI: 10.1051/e3sconf/20183302040.

Bao, C., Ma, X., Lim, K.S., Chen, G., Xu, F., Tan, F. & Abd Hamid, N.H. (2021) Seismic fragility analysis of steel moment-resisting frame structure with differential settlement. *Soil Dynamics and Earthquake Engineering*, 141, 106526. DOI: 10.1016/j.soildyn.2020.106526.

Baracous, A. (1957) The foundation failure of the Transcona Grain Elevator. *Engineering Journal*, 40, 973-977.

Dabrowski, P.S., Zienkiewicz, M.H., Truong-Hong, L. & Lindenbergh, R. (2023) Assessing historical church tower asymmetry using point cloud spatial expansion. *Journal of Building Engineering*, 75, 107040. DOI: 10.1016/j.jobe.2023.107040.

Dudek, M., Rusek, J., Tajduś, K. & Słowik, L. (2021) Analysis of steel industrial portal frame building subjected to loads resulting from land surface uplift following the closure of underground mine. *Archives of Civil Engineering*, LXVII, 283-298. DOI: 10.24425/ace.2021.138056.

Elsawwaf, A., El Sawwaf, M., Farouk, A., Aamer, F. & El Naggar, H. (2023) Restoration of tilted buildings via micropile underpinning: A case study of a multistory building supported by a raft foundation. *Buildings*, 13, 422. DOI: 10.3390/buildings13020422.

Geng, J., Meng, Z., Yin, B. & Zhu, L. (2020) Simulation on sequential construction process and structure of the Pisa Tower. *Journal of Building Construction and Planning Research*, 30-41.

Gromysz, K. (2023) Reliable tilt of objects subjected to rectification and located in mining areas. *Architecture, Civil Engineering, Environment*, 16, 79-92. DOI: 10.2478/acee-2023-0052.

Gromysz, K. (2021) Analysis of parameters of a rectified tank on the basis of in-situ tests. *Materials*, 14, 3881. DOI: 10.3390/ma14143881.

Gromysz, K. (2019a) Revitalisation of a vertically deflected historical 16th century bell tower. *IOP Conf. Ser.: Mater. Sci. Eng.*, 471, 052025. DOI: 10.1088/1757-899X/471/5/052025.

Gromysz, K. (2019b) Analysis of removal method of a 19th church's deflection. MATEC Web Conf. 284, 05005. DOI: 10.1051/matecconf/201928405005.

Gromysz, K. (2017) Methods of removing buildings deflection used in Poland. *IOP Conf. Ser.: Mater. Sci. Eng.*, 245, 032096. DOI: 10.1088/1757-899X/245/3/032096.

Halicka, A. & Zyga, J. (2019) The consequences of non-uniform founding of concrete tank in weak wet subsoil. *Studia Geotechnica et Mechanica*, 41, 263-271. DOI: 10.2478/sgem-2019-0023.

Jänichen, J., Dubois, C., Wolsza, M., Salepci, N. & Schmullius, C. (2020) Investigation of the ground motion near the Leaning Tower of Bad Frankenhausen using sentinel – 1 persistent scatterer interrerometry. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, XLIII-B3-2020, 305-312. DOI: 10.5194/isprs-archives-XLIII-B3-2020-305-2020.

Kaszowska, O., Gruchlik, P. & Mika, W. (2018) Industrial chimney monitoring – contemporary methods. *E3S Web of Conf.*, 36, 01005. DOI: 10.1051/e3sconf/20183601005.

Kijanka, M. & Kowalska, M. (2017) Inclined buildings – some reasons and solutions. *IOP Conf. Ser.: Mater. Sci. Eng.*, 245, 022052. DOI: 10.1088/1757-899X/245/2/022052.

Kowal, T. (2014) Proposal of the determination of damage in the form of permanentyl angled building shape (Propozycja ustalania wartości szkody w postaci trwałego wychylenia bryły budynku od pionu). *Przegląd Górniczy*, 164-169.

Lamich, D., Marschalko, M., Yilmaz, I., Bednářová, P., Niemiec, D., Durďák, J., Kubečka, K. & Duda, R. (2016) Utilization of engineering geology in geo-tourism: few case studies of subsidence influence on historical churches in Ostrava-Karvina District (Czech Republic). *Environ. Earth Sci.*, 75, 128. DOI: 10.1007/s12665-015-4993-3.

Liu, Y. & Liu, Z. (2008) Study on stabilization and rectification technology for inclined transmission tower. *Rock and Soil Mechanics*, 29, 173-176.

Macchi, G. (2005) Stabilization of the Leaning Tower of Pisa, In: Structures Congress 2005. Presented at the Structures Congress 2005, American Society of Civil Engineers, New York, 1-11. DOI: 10.1061/40753(171)152.

Maffei, C.E.M. & Gonçalves, H.H.S. (2016) Innovative techniques used to plumb two 57 m height concrete buildings leaning 3.8 and 3.1 %. *Innovative Infrastructure Solutions*, 1, 33. DOI: 10.1007/s41062-016-0032-9.

Orwat, J. (2020) Causes analysis of occurrence of the terrain surface discontinuous deformations of a linear type. *J. Phys.: Conf. Ser.*, 1426, 012016. DOI: 10.1088/1742-6596/1426/1/012016.

Orwat, J. & Gromysz, K. (2021) Occurrence consequences of mining terrain surface discontinuous linear deformations in a residential building. *J. Phys.: Conf. Ser.*, 1781, 012013. DOI: 10.1088/1742-6596/1781/1/012013.

Paszek, J. (2024) Analysis of the model used to predict continuous deformations of the land surface in areas subject to discontinuous deformations – a case study. *Applied Sciences*, 14, 7676. DOI: 10.3390/app14177676.

Peng, C. (2017) The application of dynamic replacement method in deviation rectification of support pile. *IOP Conf. Ser.: Earth Environ. Sci.*, 61, 012099. DOI: 10.1088/1755-1315/61/1/012099.

Rapoport, V. & Alford, J. (1989) Preloading of independent leg units at locations with difficult seabed conditions. *Marine Structures*, 2, 451-462. DOI: 10.1016/0951-8339(89)90044-0.

Ren, C. & Yan, B. (2015) Experimental research of the influence of differential settlement on the upper frame structures. Proceedings of the 3rd International Conference on Mechanical Engineering and Intelligent Systems (ICMEIS 2015). Presented at the 2015 3rd International Conference on Mechanical Engineering and Intelligent Systems, Atlantis Press, Yinchuan, China. DOI: 10.2991/icmeis-15.2015 .100.

Shi, M.L., Zhang, H. & Zhang, R.K. (2013) The inclination of bridge Pier due to neighboring embankment construction and its rectification technique. *Applied Mechanics and Materials*, 353-356, 79-83. DOI: 10.4028/www.scientific.net/AMM.353-356.79.

Strzałkowski, P. (2022) Predicting mining areas deformations under the condition of high strength and depth of cover. *Energies*, 15, 4627. DOI: 10.3390/en15134627.

Strzałkowski, P. (2019) Some remarks on impact of mining based on an example of building deformation and damage caused by mining in conditions of Upper Silesian coal basin. *Pure and Applied Geophysics*, 176, 2595-2605. DOI: 10.1007/s00024-019-02127-1.

Terracina, F. (1962) Foundations of the Tower of Pisa. *Géotechnique*, 12, 336-339. DOI: 10.1680/geot .1962.12.4.336.

Truong-Hong, L., Lindenbergh, R., Woudenberg, P., Gard, W. & Van de Kuilen, J.-W. (2021) Monitoring deformations of a wooden church tower by laser scanning. 12-th International Conference on Structural Analysis of Historical Constructions SAHC 2020, Delft University of Technology, 709-721.

Yin, H.P., Li, C.L. & Xie, Z.Y. (2011) Analysis on deviation rectification and reinforcement of buildings. *Advanced Materials Research*, 255-260, 59-64. DOI: 10.4028/www.scientific.net/AMR.255-260.59.

Yonglin, S., Lixin, W. & Zhi, W. (2010) Identification of inclined buildings from aerial LIDAR Data for disaster management. The 2010 18th International Conference on Geoinformatics, IEEE, Beijing, China, 1-5. DOI: 10.1109/GEOINFORMATICS.2010.5567852.

Yuan, G., Shu, Q., Zhang, Y., Liu, T., Ji, Y. & Xu, G. (2012) Model experiment on anti-deformation performance of a self-supporting transmission tower in a subsidence area. *International Journal of Mining Science and Technology*, 22, 57-61. DOI: 10.1016/j.ijmst.2011.07.006.

Zhang, X., Shan, R. & Lu, M. (2018) Rectification of jacking method for brick-wooden buildings in deformation analysis with CFST reinforcement. *The Structural Design of Tall Special Buildings*, 27, e1439. DOI: 10.1002/tal.1439.