**Olena GUMEN**
National Technical University of Ukraine "Kyiv Polytechnic Institute", Ukraine

**Volodymyr DOVGALIUK, Viktor MILEIKOVSKYI**
Kyiv National University of Construction and Architecture, Ukraine

# GEOMETRICAL AND KINEMATIC ANALYSIS OF TURBULENCE IN JETS FOR ENERGY EFFICIENT VENTILATION

We show an approach to estimate turbulence intensity of turbulent jet boundary layer. This approach uses geometric and kinematic analysis of simplified scheme of its macrostructure. It is a continuation of researches performed by professor of Heat Gas Supply and Ventilation Department of Kyiv National University of Construction and Architecture Andrei Tkachuk. The obtained value of maximal turbulence intensity is coincides with known experimental data. This approach allows estimating of turbulence parameters of ventilation currents for energy efficiency optimization of air exchange organization.

**Keywords:** jet boundary layer, air distribution, ventilation, turbulence intensity

## INTRODUCTION

The most important task of ventilation and air conditioning is providing optimal microclimate conditions with minimum energy consumption. The energy efficiency is very dependent on air exchange organization. The last achievements in microclimate standardization are the requirements of turbulence parameters of airflows in a working zone. During harmonization with European Norms Ukraine accepted the turbulence standardization [1]. However, imperfect theories of turbulent flow do not allow easy theoretic calculation of the parameters during ventilation system design. We can use experimental charts included in air distributors documentation or perform CFD [2] simulation. The first possibility does not take into consideration flows and objects interaction in rooms. The second option causes high time expense for 3D model (mesh) building, computing and high-cost computer equipment and software. Therefore, the development of turbulent flows theory may simplify air distribution design.

Professor of Heat Gas Supply and Ventilation Department of Kyiv National University of Construction and Architecture A. Tkachuk [3] has developed a theory of turbulent boundary layers according to singularity method. A flow is regarded as a stream of ideal liquid containing small vortex flows as "singularities". This approach allows describing the effect of vortices directly without using additional

values. Our continuation is an approach for jet flows, which describes the jet bound-ary layer as an aggregate of large-scale vortices (puffs) [4, 5]. For free flat jets it was theoretically based on tangent of jet expansion angle β [5] $\tan(\beta) = \Theta = 0.2179 \approx 0.22$. In this work we will calculate turbulent parameters of a simplest jet - free flat - using Tolmien source [4] from infinitely small slot.

## 1. MAIN CONCEPTS OF THE JET MACROSTRUCTURE ANALYSIS

In free jets [4, 5] we showed possibility of presenting the jet boundary layer (Fig. 1a) as a vortex sheet with round puffs rolling along free jet boundaries.
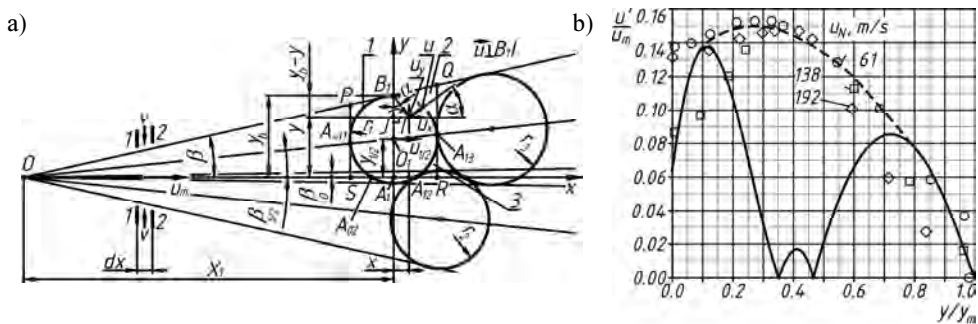


Fig. 1. Free flat Tolmien source: a) chart; b) turbulence intensity: 1 - puff; 2 - external part
of interpuff layer; 3 - internal part of interpuff layer; solid curve - calculated
by the *x*-velocity; dashed curve - interpolation augmentation by the experimental data;
squares and circles - experimental data for free jet by NASA [6] at the distance 371 mm
from round nozzle Ø 52.1 mm with nozzle velocity $\underline{u}_N$ [m/s]

A line connecting the centres of vortices $O_i$ actually coincides with the charac-teristics half-velocity line, where velocity $u$ amounts to half of axial velocity $u_m$. In the work [4] the free jet is presented as two sheets of puffs in chessboard order. The relations was obtained between the tangent $\Theta$, the tangent $\Theta_{1/2} = 0.4655\,\Theta$ of expansion angle $\beta_{1/2}$ of the half-velocity characteristics line and the tangent $\Theta_0 = 0.069\,\Theta$ of the angle $\beta_0$ of puff immersion in the adjacent layer. The correla-tion between the radius $r_i$, half-width of the jet $y_b$ and the distance $X_i$ from any (*i*-th) vortex centre $O_i$ to the slot O along the jet axis is the following:

$$r_i/y_b = (X_i/y_b)\,(\Theta - \Theta_{1/2}) = (\Theta - \Theta_{1/2})/\Theta = 1 - (\Theta_{1/2}/\Theta). \tag{1}$$

Now we will simulate turbulent pulsations around a puff 1. There are many small vortices formed around the puffs to compensate tangential velocity compo-nent rupture [3]. These and other small/medium vortices formed by puffs destroy migrate by Magnus forces and fill all of the jet volume. This migration is too random, so we need mathematical statistics to calculate corresponding pulsations.

However, to avoid it a supposition that the puffs causes main influence may be accepted.

Let us use a Euclidean coordinate system (Fig. 1): $x$ is the jet axis and axis $y$ crosses the current puff 1 centre. Ordinate of the puff 1 centre $O_1$ is $y_{1/2} = x\Theta_{1/2}$. The section width $|A_1B_1| = y_b = x\Theta$. The puff's roll [4, 5, 7] by free boundaries as wheels. In external parts 2 of interpuff layers (between puffs) flow moves only normally to the jet axis for air ejection. In internal parts there are linear [4] or parabolic [7] approximations for $x$-velocity, which take into account submerged puff parts:

$$u_x/u_m = \Pi(y/y_b) = 1.73145(y/y_b) + 1.06237 \tag{2}$$

$$u_x/u_m = \Pi(y/y_b) = -8.6222815(y/y_b)^2 + 1.7314481(y/y_b) + 1.0623752 \tag{3}$$

The puff 1 (Fig. 1) rolls on the corresponding free boundary $OQ$. So velocity $u = 0$ at the point $B_1$ of the boundary and the puff intersection.

In the introduced coordinate system the $x$-velocity [4] has the following form:

$$\frac{u_x}{u_m} = \frac{1}{2}\frac{\left(1 - (y/y_b)\right)}{1 - (\Theta_{1/2}/\Theta)} \tag{4}$$

The right branch of the puff boundary equation with boundary abscissa $x_{p.b.}$ is:

$$x_{p.b.}/y_b = \sqrt{\left(1 - (\Theta_{1/2}/\Theta)\right)^2 - \left((y/y_b) - (\Theta_{1/2}/\Theta)\right)^2} \tag{5}$$

The left branch will be noted as minus $x_{p.b.}/y_b$.

## 2. TURBULENCE INTENSITY CALCULATION

Turbulence intensity $\varepsilon$ may be calculated as turbulent pulsations $(u'^2)^{1/2}$, understood as root mean square deviation of velocity, divided by mean velocity $u$ [6, 8]:

$$\varepsilon = (u'^2)^{1/2}/\bar{u} \tag{6}$$

For 3D flows we can use quadratic mean of velocity components $(u'^2_x)^{1/2}$, $(u'^2_y)^{1/2}$ and $(u'^2_z)^{1/2}$ as $(u'^2)^{1/2}$ as it is described at CFD Online Book (http://www.cfd-online.com/Wiki/Turbulence_intensity):

$$(u'^2)^{1/2} = (u'^2_x + u'^2_y + u'^2_z)^{1/2}$$

In jets the main velocity component is the $x$-component. So in the work [8] authors used only the $x$-component of turbulence intensity in the equation (6):

$$\varepsilon_x = (u'^2_x)^{1/2}/\bar{u} \tag{7}$$

The next problem is to calculate the root mean square. Let us consider the following task. We have a value $v$ that changes in the time $t$. We assumed it as a random value, measured it $n$ times and found probability momentums incl. mean $\bar{v}$ and root mean square:

$$v' = \sqrt{\sum (v - \bar{v})^2 / (n-1)} \qquad (8)$$

There was many researches performed, many relations between the value $v$ and other parameters (excluding the time) was found and used in practice. Finally we discovered that this value $v$ is actually not random and found a very complex dependency $v = f(t)$. It is not easy to calculate $f$. Nevertheless, can we use the relations that was found previously by the assumption that it is random? The only possible answer is "yes" because the last discovery did not change anything in the nature of $v$. There is only one difference that we can obtain infinite number of value $v$ by the function calculation. So we can integrate the equation (8) using $n = \infty$. At this case minus one may be cancelled and we will obtain simple mean of square of deviation may be replaced by the following using starting time $t_0$

$$v' = \sqrt{\int_{t_0}^{t} (v - \bar{v})^2 \, dt / (t - t_0)} \qquad (9)$$

By the same way we can find other probability momentums and refine or prove the previously found experimental relations.

Using the equations (9) and substituting $v$ by the velocity pulsation $u'^2_x$ we will find them. In the previous works [4] we successfully used integration by the abscissa $x$ from $A_1$ ($x = 0$) to $R$ ($x = r_1$) instead of integration by the time. This range is because of the symmetry of $x$-velocity in a puff with respect to the $y$-axis. Integrations may be performed separately by the puff and the interpuff layer. Mean value by integration of the equations (1)-(5):

$$\bar{u}_x / u_m = \frac{1}{r_1/y_b} \int_0^{r_1/y_b} \frac{u_x}{u_m} d\frac{x}{y_b} = \frac{1}{r_1/y_b} \left( \int_0^{x_{p.b.}/y_b} \frac{u_x}{u_m} d\frac{x}{y_b} + \int_{x_{p.b.}}^{r_1/y_b} \frac{u_x}{u_m} d\frac{x}{y_b} \right) =$$

$$= \begin{cases} \dfrac{1}{2} \dfrac{1 - (y/y_b)}{1 - (\Theta_{1/2}/\Theta)} S & \text{at} \quad y/y_b \geq \Theta_{1/2}/\Theta; \\[2mm] \dfrac{1}{2} \dfrac{1 - (y/y_b)}{1 - (\Theta_{1/2}/\Theta)} S + \Pi(y/y_b)(1-S) & \text{at} \quad y/y_b < \Theta_{1/2}/\Theta, \end{cases} \qquad (10)$$

where $S$ - parameter:

$$S = \sqrt{1 - \left( \frac{(y/y_b) - (\Theta_{1/2}/\Theta)}{1 - (\Theta_{1/2}/\Theta)} \right)^2} \qquad (11)$$

The velocity pulsation by the equations (2)-(5), (7), (9), (10) and (11)

$$
\frac{u'_x}{u_m} = \sqrt{\frac{1}{r_1/y_b}\left(\int_0^{x_{p.b.}/y_b}\left(\frac{u_x}{u_m}-\frac{\bar{u}_x}{u_m}\right)^2 d\frac{x}{y_b} + \int_{x_{p.b.}/y_b}^{r_1/y_b}\left(\frac{u_x}{u_m}-\frac{\bar{u}_x}{u_m}\right)^2 d\frac{x}{y_b}\right)} =
$$

$$
= \begin{cases} \dfrac{1}{2}\dfrac{1-(y/y_b)}{1-(\Theta_{1/2}/\Theta)}\sqrt{S(1-S)} & \text{at} \quad y \geq X_1\Theta_{1/2}; \\ \left|\dfrac{1}{2}\dfrac{1-(y/y_b)}{1-(\Theta_{1/2}/\Theta)}-\Pi\left(\dfrac{y}{y_b}\right)\right|\sqrt{S(1-S)} & \text{at} \quad y < X_1\Theta_{1/2}. \end{cases} \quad (12)
$$

The result plot (Fig. 1b) by the equations (12) contains the *x*-component (solid) chart that has two maximums. The curve depression is caused by ignoring of the *y*-components and small-scale vorticity so we build the interpolation line (dashed) from left maximum to a point near to the second maximum keeping smooth curve. Nevertheless, the solid line shows the maximal turbulence intensity near to 14% but the real value [6] is approximately 15%. The difference of 1% is very small so the geometrical analysis can forecast the maximal turbulence intensity.

Further research will be focused on the geometric and kinematic analysis of the turbulence intensity in jets, which are laid on different shape surfaces. This approach may be helpful for energy efficient air exchange organization.

## CONCLUSIONS

1. An improved analytical description of turbulence intensity in free flat jets without the use of experimental values is suggested based on geometric and kinematic analysis of their macrostructure. This approach may be helpful for energy-efficient air exchange organization development complaint with last European standards.
2. The approach shows that the maximal turbulence intensity in free jets is approx. 14%. It differs from known experimental data only for 1%. Therefore, the result corresponds with the experimental research data.

## REFERENCES

[1] ДБН В.2.5.-67:2013. Опалення, вентиляція та кондиціонування, Мінрегіон України, Укрархбудінформ, К.: 2013, 142 с.
[2] Thool S.B., Sinha S.L., Simulation of room airflow using CFD and validation with experimental results, International Journal of Engineering Science and Technology 2014, 6, 5, 192-202.
[3] Ткачук А.Я., Довгалюк В.Б., Аеродинаміка вентиляції: Навчальний посібник, ІВНВКП «Укргеліотех», 2009, 376 с.
[4] Мілейковський В.О., Геометричне обґрунтування профілю швидкості в струминних примежових шарах, Будівництво України: Науково-виробничий журнал 2010, № 1, с. 17-20.

[5]  Gumen O., Dovhaliuk V., Mileikovskyi V., Geometric modelling of turbulent flat jets in accompanying and contrary flows, Budownictwo o Zoptymalizowanym Potencjale Energetycznym 2015, No 1(15), s. 70-77.

[6]  Boldman D.R., Brinich P.F., Mean velocity, turbulence intensity, and scale in a subsonic turbulent jet impinging normal to a large flat Plate, NASA Technical Paper 1037 [Electronic resource], USA, NASA, 1977, p. - 34 p. [Access mode http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/ 19770025487.pdf]

[7]  Гумен О.М., Довгалюк В.Б., Мілейковський В.О., Визначення кількості руху струминних течій, Науково-технічний збірник "Енергоефективність в будівництві та архітектурі", Випуск 6. Відповідальний редактор П.М. Куліков, КНУБА, К.: 2014, с. 68-75.

[8]  Juodis E.S., Motuzienė V., Vėdinimo aerodinamika: vadovėlis, Technika, Vilnius 2014, 296 c.

## GEOMETRYCZNA I KINEMATYCZNA ANALIZA TURBULENTNEGO PRZEPŁYWU STRUMIENIA DLA UZYSKANIA EFEKTYWNEJ ENERGETYCZNIE WENTYLACJI

Przedstawiono podejście do oceny intensywności przepływu powietrza w turbulentnej warstwie granicznej. Podejście to wykorzystuje geometryczną i kinematyczną analizę uproszczonego schematu jej makrostruktury. Zaprezentowana analiza jest kontynuacją badań przeprowadzonych przez profesora Wydziału Ciepła i Wentylacji Kijowskiego Narodowego Uniwersytetu Budownictwa i Architektury Andrzeja Tkaczuka. Otrzymana wartość maksymalnej intensywności turbulentnego przepływu jest zbieżna ze znanymi już danymi eksperymentalnymi. Podejście to pozwala na oszacowanie parametrów turbulentnego przepływu powietrza w wentylacji w celu optymalizacji pod kątem jej efektywności energetycznej.

**Słowa kluczowe:** warstwa graniczna przepływu, dystrybucja powietrza, wentylacja, intensywność turbulentnego przepływu powietrza