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The optimal insulation thickness determined for modular buildings according to multi-criteria analysis

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Abstract: The possibility of using modular buildings for residential and public needs is considered. Due to the peculiarities of technology and the use of thermal insulation materials, such structures are erected in a short time with the help of ready-made modules, characterized by their simplicity and speed of installation. The results of the multi-criteria analysis are presented in the form of petal diagrams and diagrams of global priorities.

Keywords: thermal insulation, polyurethane foam, modular construction, petal diagrams

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Introduction

Currently, there is interest in the possibility of creating inexpensive and comfortable housing, which is affordable and can be widely implemented by the construction industry, while also taking into account modern conditions. Thus, existing scientific research continues its work in defining the most constructive schemes and methods for transforming modular volume components, taking into account natural-climatic, as well as social and national features of regions (Marza et al., 2019; Samoilovych & Orlova, 2016).

Modular construction is gaining popularity in the building of production facilities (Huang & Krawczyk, 2007; Khliupin & Kravchuk, 2018). Such buildings meet modern requirements for industrial construction, in particular, they have an attractive appearance, a high-quality load-bearing and energy-saving structure of external fences, can be constructed in the shortest possible time and have low operating costs. This is all made possible through the use of light metal structures (Avdeeva & Kaplina, 2015; Bondarenko, 2011; Lisenko et al., 2006). Such construction has become widespread due to the transportability of these steel structures, a high degree of factory readiness for load-bearing and enclosing elements, as well as the ability to move the building to a new site.

1. The purpose of the work

The theoretical substantiation of the optimal thermal insulation thickness for the external protections of modular buildings, taking into account evaluation criteria based on the method of analysis of hierarchies.

Multi-criteria decision-making, in contrast to analysis based on a single criterion, helps to create a holistic set of criteria that will function as a tool for full evaluation and will formulate, use and transform benefits in the decision-making process. Thus, when justifying the decision on the choice of thermal insulation thickness, which can be integrated with energy efficiency and environmental protection and for which many problems need to be solved, it is strongly recommended to use multi-criteria decision-making methods. When quantifying the criterion of system activity by a set of parameters, it is necessary to carry out a hierarchical representation of the influencing factors. This uses the method of analysis of hierarchies, which is one of the ways to conduct complex examinations. The method of analysis of hierarchies involves the decomposition of the problem into simpler components.

2. Conducting multi-criteria analysis of options using pairwise comparisons

Starting the evaluation of insulating material thickness, it is necessary to solve two problems (Fig. 1):

1. Form a group of experts
2. Develop a system of criteria by which the analysis will be conducted.

Five experts were chosen, all of them independent, qualified specialists, who have experience in this field.

The following criteria were selected in order to evaluate the options:

C_1 – Cost; C_2 – Thermal conductivity resistance; C_3 – Weight; C_4 – Operating costs;

$C = \{C_1, C_2, C_3, ..., C_8\}$.

Several thicknesses of thermal insulation are used for the construction of modular buildings: Polyurethane foam – 100 mm – S_1, Polyurethane foam – 120 mm – S_2, Polyurethane foam – 150 mm – S_3, Polyurethane foam – 200 m – S_4. All these options are subject to multi-criteria analysis and it is necessary to arrange the elements of the set S according to the criteria from set C.

The membership function of fuzzy sets were determining based on expert information by pairwise comparisons of variants using the 9-point Saati scale (Krot, 2017).

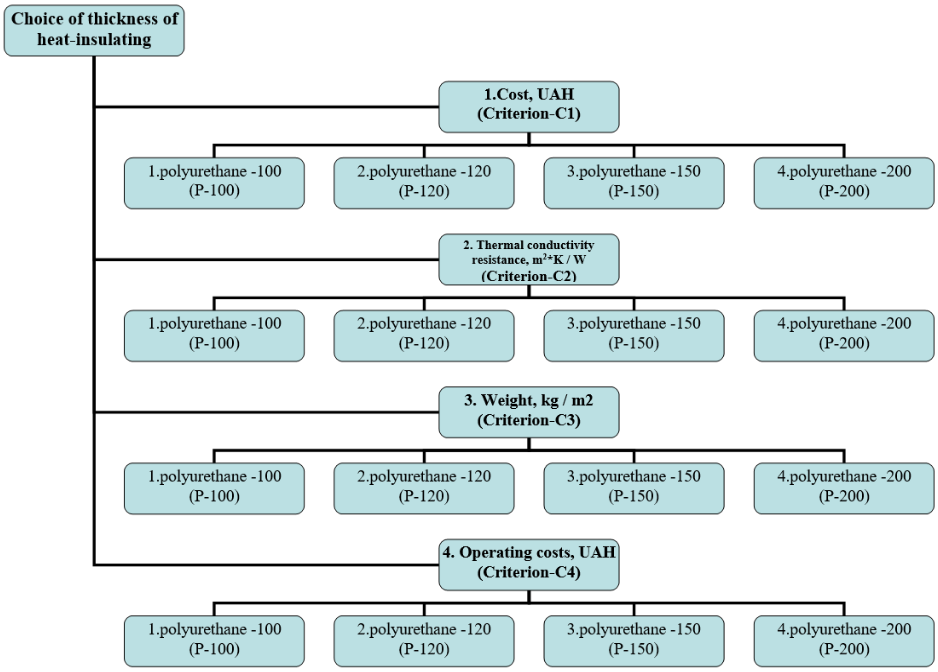

Fig. 1. Hierarchy of the optimization problem of choosing the thickness of thermal insulation (*own study*)

Matrices of pairwise comparisons filled in by experts are presented in Table 1.

Table 1. Matrices of pairwise comparisons (*own study*)

Selected criteria	Matrices of pairwise comparisons
Criterion C_1: Cost, UAH	$A(C_1)=\begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0.5 & 1 & 4 & 5 \\ 0.33 & 0.25 & 1 & 5 \\ 0.25 & 0.2 & 0.2 & 1 \end{bmatrix}$ $AC_1=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0.5 & 1 & 4 & 5 \\ 0.33 & 0.25 & 1 & 5 \\ 0.25 & 0.2 & 0{,}2 & 1 \end{pmatrix}$
Criterion C_2: Thermal conductivity resistance, $m^2 \cdot K/W$	$A(C_2)=\begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array}\begin{bmatrix} 1 & 0.5 & 0.2 & 0.12 \\ 2 & 1 & 0.25 & 0.2 \\ 5 & 4 & 1 & 0.25 \\ 8.33 & 5 & 4 & 1 \end{bmatrix}$ $AC_2=\begin{pmatrix} 1 & 0.5 & 0.2 & 0.12 \\ 2 & 1 & 0.25 & 0.2 \\ 5 & 4 & 1 & 0.25 \\ 8.33 & 5 & 4 & 1 \end{pmatrix}$
Criterion C_3: Weight, kg/m^2	$A(C_3)=\begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array}\begin{bmatrix} 1 & 1.11 & 2 & 4 \\ 0.9 & 1 & 3 & 4 \\ 0.5 & 0.3 & 1 & 4 \\ 0.25 & 0.3 & 0.25 & 1 \end{bmatrix}$ $AC_3=\begin{pmatrix} 1 & 1.11 & 2 & 4 \\ 0.9 & 1 & 3 & 4 \\ 0.5 & 0.3 & 1 & 4 \\ 0.25 & 0.3 & 0.25 & 1 \end{pmatrix}$
Criterion C_4: Operating costs, UAH	$A(C_4)=\begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \end{array}\begin{bmatrix} 1 & 0.4 & 0.2 & 0.12 \\ 2.5 & 1 & 0.3 & 0.25 \\ 5 & 3.3 & 1 & 3.5 \\ 8.33 & 4 & 2.86 & 1 \end{bmatrix}$ $AC_4=\begin{pmatrix} 1 & 0.4 & 0.2 & 0.12 \\ 2.5 & 1 & 0.3 & 0.25 \\ 5 & 3.3 & 1 & 3.5 \\ 8.33 & 4 & 2.86 & 1 \end{pmatrix}$

The principle of synthesis of priorities is to develop a global criterion based on a system of local criteria. Local criteria are defined as the priority vectors of each matrix of pairwise comparisons (Saati, 1993).

The eigenvector of the matrix is denoted

$$AC_{11} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}, \tag{1}$$

where: a_0, a_1, a_3 … a_n – the value of the components of the eigenvector of the matrix. For criterion C_1 (material cost), the matrix of pairs is calculated below. Matrices of pairwise comparisons for other criteria are calculated similarly.

$$\begin{aligned} AC_{1\,0.1} = 2 \quad AC_{1\,0.2} = 3 \quad AC_{1\,0.3} = 4 \\ AC_{1\,1.2} = 4 \quad AC_{1\,1.3} = 5 \\ AC_{1\,2.3} = 5 \end{aligned} \tag{2}$$

$$AC_1 = \begin{pmatrix} 1 & AC_{1\,0.1} & AC_{1\,0.2} & AC_{1\,0.3} \\ \dfrac{1}{AC_{1\,0.1}} & 1 & AC_{1\,1.2} & AC_{1\,1.3} \\ \dfrac{1}{AC_{1\,0.2}} & \dfrac{1}{AC_{1\,1.2}} & 1 & AC_{1\,2.3} \\ \dfrac{1}{AC_{1\,0.3}} & \dfrac{1}{AC_{1\,1.3}} & \dfrac{1}{AC_{1\,2.3}} & 1 \end{pmatrix} \tag{3}$$

$$AC_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0.5 & 1 & 4 & 5 \\ 0.33 & 0.25 & 1 & 5 \\ 0.25 & 0.2 & 0.2 & 1 \end{pmatrix} \tag{4}$$

To determine the geometric mean for each row of the matrix of even comparisons, the following is used:

$$AC_{11} = \begin{pmatrix} \sqrt[4]{AC_{1\,0.0} + AC_{1\,0.1} + AC_{1\,0.2} + AC_{1\,0.3}} \\ \sqrt[4]{AC_{1\,1.0} + AC_{1\,1.1} + AC_{1\,1.2} + AC_{1\,1.3}} \\ \sqrt[4]{AC_{1\,2.0} + AC_{1\,2.1} + AC_{1\,2.2} + AC_{1\,2.3}} \\ \sqrt[4]{AC_{1\,3.0} + AC_{1\,3.1} + AC_{1\,3.2} + AC_{1\,3.3}} \end{pmatrix} AC_{11} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} AC_{11} = \begin{pmatrix} 2.213 \\ 1.778 \\ 0.803 \\ 0.316 \end{pmatrix} \tag{5}$$

Add the elements of this column:

$$\sum AC_{11} = 5.111 \tag{6}$$

Next, it is necessary to determine the vector of local priorities (AC_{111}), which will show the importance of the compared criteria from the point of view of this expert. The component of the priority vector is defined as the ratio of the component of the eigenvector of the matrix to the sum of the values of its components (Saati, 1991; Saati, 1993).

To determine the relationship of consistency:

$$OC = \frac{IC}{CC} \leq 0.2, \tag{7}$$

where:
OC – the consistency ratio,
IC – consistency index,
CC – the value corresponding to the average random consistency of the matrix of this order, is determined by Table 2.

Table 2. Average consistencies of random matrices (*own study*)

The size of the matrix	Random consistency
1.2	0
…	….
9	1.45

The consistency index can be determined by the following formula:

$$IC = \frac{\lambda_{max} - n}{n - 1}, \tag{8}$$

where:
n – the number of elements being compared,
λ_{max} – the calculated value.

To calculate λ_{max}, the sum of each column of the matrix is determined, which is multiplied by the corresponding component of the priority vector. Conventionally, this can be represented as follows (Saati, 1991; Saati, 1993):

$$\sum S_1 * x_1 + \sum S_2 * x_2 + \sum S_3 * x_3 + ... + \sum S_n * x_n = \lambda_{max}, \tag{9}$$

where: $\sum S_1$, $\sum S_2$, $\sum S_3$, $\sum S_N$ – the sum of the elements of the corresponding columns of the matrix.

$$AC_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0.5 & 1 & 4 & 5 \\ 0.33 & 0.25 & 1 & 5 \\ 0.25 & 0.2 & 0.2 & 1 \end{pmatrix}$$

$$\sum 1 = \left(\sum AC_1^{<0>} \sum AC_1^{<1>} \sum AC_1^{<2>} \sum AC_1^{<3>} \right)$$

$$\sum 1 = \begin{pmatrix} 2.08 & 3.45 & 8.2 & 15 \end{pmatrix}$$

So, in our case, the calculated value for the criterion C_1: $\lambda_{max} = \lambda_1 = 4.319$.
Number of elements (options) being compared, $n = 4$.

$$IC = \frac{4.319 - 4}{4 - 1} = 0.106. \tag{10}$$

Random consistency for the size of the matrix "4" – Table 2: $CC_1 = 0.9$.
Consistency ratio:

$$OC = \frac{IC}{CC} = \frac{0.106}{0.9} = 0.12 \le 0.2.$$

The obtained values of the priority vector (AC_{111}) is a system of local criteria, based on which, the global priority of the variant for each variant is calculated (Saati, 1991; Saati, 1993).

$$P_{j\text{r}} = \sum_{i=1}^{m} P_j(i) \cdot w(i), \tag{11}$$

where:
$P_{j\text{r}}(i)$ – the priority of the *j-th* alternative according to the *i-th* criterion,
$w(i)$ – the significance of the *i-th* criterion.
To select the insulation thickness, the local priority vectors are calculated:

$$\text{P\_g} = AC_{111} \cdot W_1 + AC_{121} \cdot W_2 + AC_{131} \cdot W_3 + AC_{141} \cdot W_4$$

Vector of global priorities:

$$\text{P\_g} = \begin{pmatrix} 0.904 \\ 0.936 \\ 0.871 \\ 1.289 \end{pmatrix} \begin{matrix} S_1 & \text{polyurethane} - 100\text{ mm} \\ S_2 & \text{polyurethane} - 120\text{ mm} \\ S_3 & \text{polyurethane} - 150\text{ mm} \\ S_4 & \text{polyurethane} - 200\text{ mm} \end{matrix}$$

Option S_4 has the highest global priority. The values of the vectors of local priorities for choosing the type of thermal insulation is in Table 3.

Table 3. Vectors of local priorities for selection of thermal disposal facilities (*own study*)

Name of the criterion	Vector of local priorities	Consistency ratio (OC)	Significance (weight) of the criterion ($w(i)$)
Criterion C_1: Cost, UAH	$AC_{111} = \begin{pmatrix} 0.433 \\ 0.348 \\ 0.157 \\ 0.062 \end{pmatrix}$	0.12	1.0
Criterion C_2: Thermal conductivity resistance $m^2 \cdot K/W$	$AC_{121} = \begin{pmatrix} 0.055 \\ 0.094 \\ 0.250 \\ 0.601 \end{pmatrix}$	0.07	1.0
Criterion C_3: Weight, kg/m^2	$AC_{131} = \begin{pmatrix} 0.360 \\ 0.378 \\ 0.188 \\ 0.074 \end{pmatrix}$	0.04	1.0
Criterion C_4: Operating costs, UAH	$AC_{141} = \begin{pmatrix} 0.055 \\ 0.116 \\ 0.275 \\ 0.553 \end{pmatrix}$	0.04	1.0

Interpretation of the analysis results, implemented in the form of a petal diagram (Fig. 2). Diagram of global priorities is presented in Figure 3.

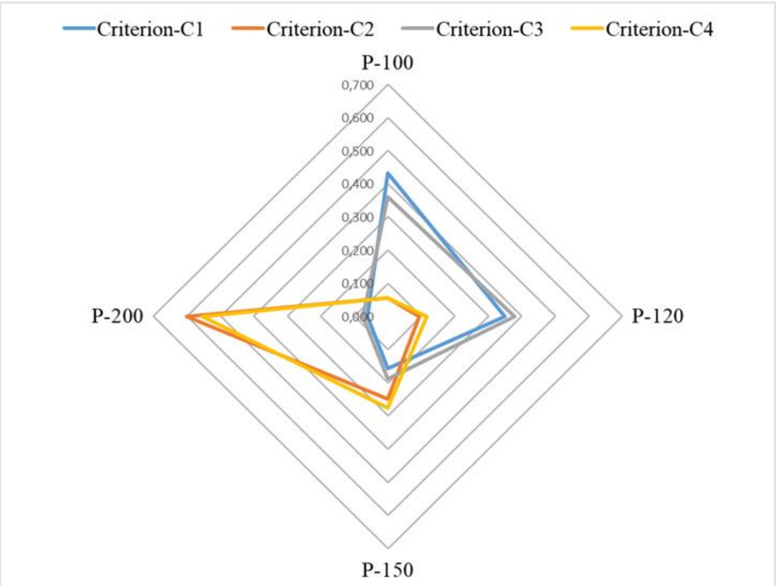

Fig. 2. The results of multi-criteria analysis for choosing the type of thermal insulation material – petal diagram (*own study*)

The diagram shows that for polyurethane foam with a thickness of 200 mm, the highest value is the high thermal conductivity and low operating costs. In turn, polyurethane foam with a thickness of 100 mm has low capital costs and low specific weight.

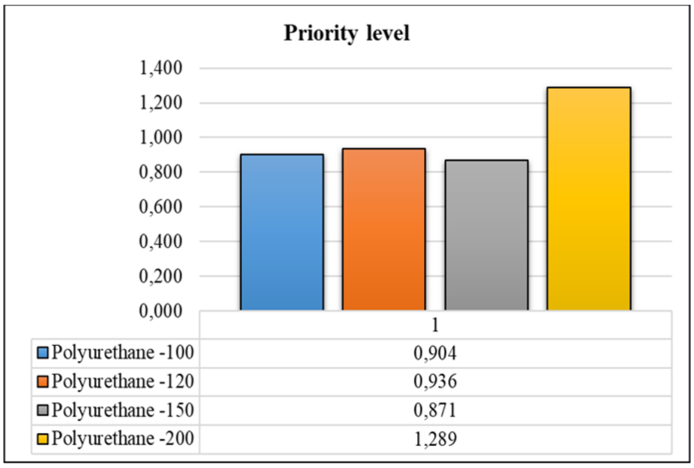

Fig. 3. Diagram of global priorities (*own study*)

Taking into account the value of global priorities from the diagram (Fig. 3) it is seen that the most optimal thickness of the insulating material is polyurethane foam 200 mm.

Conclusions

The possibility of using modular buildings for residential and public needs is considered. Due to the peculiarities of technology and the use of thermal insulation materials, such structures are erected in a short time with the help of ready-made modules, which are characterized by simplicity and speed of installation.

The choice of the thickness of the thermal insulation material for the external protection of the modular building was based on the application of a multi-criteria analysis of different thicknesses of thermal insulation.

The use of petal diagrams and global priority diagrams made it possible to make an in-depth assessment of several thicknesses of thermal insulation. It should be pointed out that the importance of each of the criteria was taken into account for the selection of thermal insulation.

As a result of the created system of expert assessments to determine the thickness of thermal insulation, which can be used for light prefabricated structures of modular buildings, based on multi-criteria analysis of options using pairwise comparisons, it was found that the most suitable insulation material is 200 mm thick polyurethane. The results of the multi-criteria analysis are presented in the form of petal diagrams and diagrams of global priorities.

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