



Investigating multi-objective time, cost, and risk problems using the Grey Wolf Optimization algorithm

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Abstract: Safety plays a crucial role in construction projects. Safety risks encompass potential hazards such as work accidents, injuries, and security. Consequently, it is important to effectively manage these risks with equal emphasis on time and cost considerations during the project planning phase. Within the scope of this research, the grid and archive-based Grey Wolf Optimizer (GWO) algorithm was employed to investigate multi-objective time-cost-risk problems. By employing the GWO, multiple Pareto solutions were provided to the decision-maker, facilitating improved decision-making. It was determined that the GWO algorithm yields better results in time-cost-risk problems compared to the Particle Swarm Optimization (PSO) and Differential Evolution (DE) algorithms.

Keywords: multi-objective optimization, grey wolf optimization algorithm, time-cost-risk

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Introduction

Construction projects are complex processes that require the consideration of multiple factors. During the planning stage of these projects, in addition to time and cost, risk factors should also be taken into account as a fundamental consideration. Construction projects involve complex processes that entail a series of uncertainties

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and risks. These risks can lead to financial losses, delays, and even complete project failure. Therefore, the relationship between time, cost, and risk is of great importance in the planning and management of projects. A review of the literature shows that various metaheuristic algorithms are used to solve time-cost trade-off problems. Some of these metaheuristic approaches include the Genetic Algorithm (GA) (Sonmez & Bettemir, 2012; Zheng et al., 2005), Rao Algorithm (RA) (Yilmaz & Dede, 2023), Particle Swarm Optimization (PSO) (Aminbakhsh & Sonmez, 2017; Zhang & Li, 2010), Teaching-learning based optimization (TLBO) (Mohammadi et al., 2022) and other similar methods. In addition to time-cost, many studies have expanded the literature by adding objectives such as quality risk and environmental impact (Kaveh et al., 2021; Ozcan-Deniz et al., 2012; Sharma et al., 2023). The aim of this study was to provide decision-makers with multiple options in order to understand the relationships between time, cost, and risk in construction projects and to investigate the applicability of metaheuristic algorithms for optimizing these factors. Within this scope, a multi-objective time-cost-risk problem was examined using a grid and archive-based grey wolf optimization algorithm.

1. Problem formulation

Considering the number ways a project may be approached, completion time, cost, and project safety risk can vary greatly. As a result, the approach to the project must be carefully chosen. Within the scope of this study, the input parameters considered for the exchange optimization model included completion time, cost, and safety risk. The objective was to simultaneously optimize the duration, total cost, and risk of the project

$$\min(T, C, R) \quad (1)$$

2. Multi objective optimization

Within the literature, several methodologies have been proposed to address multi-objective problems. Knowles & Corne (2000) proposed an approach that utilizes an external archive and adaptive grids: the Pareto Archived Evolution Strategy (PAES). The newly generated population is incorporated into the archive if it outperforms the existing population. In the case where the number of archived solutions exceeds the initially specified threshold, the adaptive grid mechanism is activated to perform elimination. The primary objective of this operator is to maximize the diversity among the Pareto optimal solutions. In the present study, the Grey Wolf Algorithm, as proposed by Mirjalili et al. (2016), is implemented using the operators employed by the Pareto Archived Evolution Strategy (PAES) to attain Pareto solutions.

2.1. Grey Wolf Optimization

The grey wolf optimization algorithm is inspired by the hunting and hierarchy of gray wolves. Alpha (α), beta (β), delta (δ) and omega (ω) gray wolves are located from the top to the bottom in the hierarchy (Mirjalili et al., 2016).

In the grey wolf optimization algorithm, used in multi-objective time-cost-risk problems, a grid is created by dividing the search space formed by the populations in the archive into equal sized regions. The aim is to select leaders from the least dense region among these defined regions using a roulette wheel probability mechanism. When the number of archives exceeds the initially specified limit, populations from the most crowded region within the grid are randomly removed until the archive count is reduced to the maximum allowed number. The pseudocode for obtaining the pareto solutions of the multi-objective time cost risk problem within the grey wolf algorithm is presented in Figure 1.

Pareto solutions of the multi-objective time cost risk problem with GWO

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1: Initialization Parameters (Number of Populations (N), Iterations (it), Number of
Grids (Grid), Number of Archives (NA))
2: Initialize  $X_i$  ( $i=1, \dots, N$ )
3: Cost Function ( $i$ )= $f(X(i))$  (Eq.1)
4:  $P_1$ = Find the non-dominated solutions and initialized the archive with them
5: for  $i=1$ : iteration
6:  $X_\alpha, X_\beta, X_\delta$ ; Select Leader(Archive)
7: for  $j=1$ : N
8:      $X_1(i), X_2(i), X_3(i)$  calculation
9:      $X(i) = (X_1(i) + X_2(i) + X_3(i)) / 3$ 
10: end for(7)
11:  $P_{new}$ = Find the non-dominated solutions ( $[f(X(i))]$ )
12:  $P_1$ = Identify non-dominated populations and update archive ( $[P_1; P_{new}]$ )
13: if Number of populations in the archive  $> NA$ 
14:     Run the grid mechanism to omit one of the current archive members
15: end if (13)
16: end for (5)

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Fig. 1. Pseudo code of the multi-objective time cost risk problem with GWO algorithm
(*own research*)

3. Numerical examples

This study evaluated the efficacy of the grey wolf algorithm in addressing multi-objective problems involving time, cost, and risk using two small-scale construction projects. In the context of multi-objective construction scheduling, the selected examples included a project with eleven activities and six objectives, as well as a problem with thirteen activities and four objectives encompassing time, cost, quality, and risk. This research was primarily focused on addressing the objectives pertaining to time, cost, and risk in both scenarios, while ignoring other objectives.

3.1. Eleven Activity Time-Cost-Risk Problem

The original formulation of the problem was proposed by Ozcan-Deniz et al. (2012), considering time, cost, and environmental impact as the main objectives. Kaveh et al. (2021) further extended these objectives by incorporating quality, risk, and resource utilization for each activity option. In this study, only the risk options suggested by Kaveh et al. (2021) were considered, transforming the problem into a multi-objective time-cost-risk problem. The details of the problem are presented in Table 1. Kaveh et al. (2021) solved the problem using a non-dominant ranking-based reference point approach with a differential evolution algorithm, employing 100 populations and 200 iterations.

Table 1. Eleven Activity Time-Cost-Risk Option (*own research*)

ID	Activity	Successors	Alt.	Time [Days]	Cost [\$]	Risk	
						RL	RS
1	Site work	2	1	4	5039.71	2	2
			2	4	4924.93	2	3
2	Excavation	3	1	2	360.71	3	3
			2	2	297.05	4	4
3	Footing	4	1	6	84232.67	3	5
			2	5	90392.28	4	6
4	Stem wall	5	1	13	76650.79	2	5
			2	8	86174.94	3	6
5	Slab	6	1	11	14636.05	3	4
			2	7	16758.59	2	5
6	Exterior wall	7	1	6	25959.52	4	5
			2	14	65399.94	1	5
			3	5	127542.42	3	5
7	Interior wall	11	1	18	27970.53	1	3
			2	10	35650.22	2	2
			3	15	27508.21	1	4
			4	8	34365.99	3	4
8	Flooring	–	1	16	28341.6	2	5
			2	12	45616.48	2	4
			3	8	36554.88	3	5
9	Exterior finish	–	1	31	69659.78	2	2
			2	23	233034.5	3	3
10	Interior finish	–	1	3	4006.8	1	2
			2	4	1746.55	1	3
11	Roof	8,9,10	1	21	117851.84	3	6
			2	23	69253.17	4	5

RL – risk likelihood, RS – risk severity

In this study, the GWO algorithm was employed with 100 populations and 100 iterations to obtain a solution. The hypervolume value, was calculated as 0.9495 for GWO. A comparison between some of the Pareto solutions obtained in this study and those presented by Kaveh et al. (2021) is presented in Table 2. The obtained results were compared with the results obtained using DE, and this comparison is illustrated in Figure 2.

Table 2. Results of GWO and DE (*own research*)

Algorithm	Variant	DE			GWO			Selected Options
		Time	Cost	Risk	Time	Cost	Risk	
1	Min. time	83	743750	133	83	761144.26	129	1,1,2,2,2,3,4,2,2,1,1
2		84	659560	134	84	735334.74	125	2,1,1,2,2,3,4,1,2,2,1
3		85	650960	135	85	712545.59	131	1,1,2,2,2,3,4,2,2,1,2
4	Min. cost	111	405650	110	107	405511,3	109	1,1,1,1,2,1,3,1,1,2,2
5		114	412060	115	110	408233,87	107	1,1,1,1,2,1,1,1,1,1,2
6		107	415980	113	106	411670,91	118	1,1,2,1,2,1,3,1,1,2,2
7	Min. risk	116	511290	89	113	513085,52	89	1,1,1,1,2,2,3,2,1,1,1
8		110	455350	93	113	510825,27	90	1,1,1,1,2,2,3,2,1,2,1
9		122	445550	94	110	472628,86	91	1,1,1,1,2,2,2,2,1,1,2

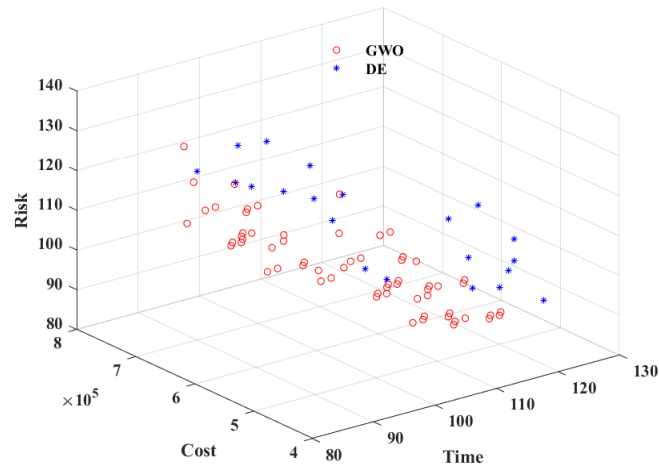


Fig. 2. DE solutions dominated by GWO (*own research*)

3.2. Thirteen Activity Time-Cost-Risk Problem

The original formulation of the problem was addressed by Sharma et al. (2023) using a four-objective approach and solved using the multi-objective Particle Swarm Optimization. The MOPSO algorithm was utilized to obtain a solution with 100 populations and 100 iterations, and the Pareto solutions satisfying the criteria

of less than 200 days and \$650,000 were presented to the decision maker. Additionally, within the scope of this study, which defined the problem as time-cost-risk, a solution was obtained using the Grey Wolf Algorithm with 100 populations and 100 iterations. The details of the problem are presented in Table 3.

Table 3. Thirteen Activity Time-Cost-Risk Option (*own research*)

ID	Activity	Successors	Alt.	Time [Days]	Cost [\$]	Risk-1		Risk-2		Risk-3	
						RL	RS	RL	RS	RL	RS
1	Site clearance	2	1	8	10039.42	1	1	1	1	1	1
			2	8	9849.86	1	2	1	2	1	2
2	Excavation	3	1	6	1082.13	3	3	1	1	3	2
			2	6	891.05	3	4	2	1	4	3
3	Footing	4	1	12	15545.67	3	2	4	2	3	2
			2	10	17039.34	2	2	3	3	3	2
4	Formwork	5	1	5	562.13	3	5	2	1	2	2
			2	4	590.32	2	4	3	2	3	4
5	Retaining wall	6	1	26	15834.49	3	3	2	2	1	2
			2	16	17274.94	2	4	3	3	1	1
6	Basement	7	1	32	74124.65	2	3	4	5	3	3
			2	29	76345.78	3	4	4	6	4	4
			3	23	84312.34	4	5	5	6	3	3
7	Slab	8	1	22	32646.05	4	5	3	3	3	4
			2	11	29759.59	4	6	4	4	4	4
8	Exterior wall	9	1	18	65959.52	2	3	2	4	2	3
			2	29	105296.94	2	2	3	3	1	2
			3	11	157433.42	2	2	3	2	2	3
9	Interior wall	13	1	37	58570.35	2	3	2	4	2	1
			2	21	59999.39	2	2	3	3	1	2
			3	32	57668.29	2	2	3	2	2	2
			4	17	63321.11	2	2	3	3	2	3
10	Flooring	-	1	34	38411.50	2	3	2	4	2	1
			2	17	65326.48	2	2	3	3	1	2
			3	12	50214.22	2	2	3	2	2	3
11	Exterior finish	-	1	9	12216.23	2	5	2	2	2	3
			2	12	3846.23	3	6	3	3	3	3
12	Interior finish	-	1	41	90219.78	1	2	2	3	1	3
			2	31	233034.50	2	3	3	3	2	3
13	Roof	10,11,12	1	23	127641.84	2	3	3	4	2	4
			2	24	81323.17	3	4	4	5	3	5

The obtained results were compared with the results obtained using MOPSO, and this comparison is illustrated in Figure 3. Table 4 displays the comparisons among Pareto solutions. The performance measurement for the multi-objective optimization algorithm was evaluated using the Hypervolume value, which was calculated as 0.79 for MOPSO and 0.85 on average for GWO. According to the Hypervolume (HV) value, even though the MOPSO algorithm solved an 11-activity problem for four objectives, the GWO algorithm achieved significantly better results, even for a three-objective problem, with fewer function evaluations. This outcome demonstrates the effective performance of the GWO algorithm.

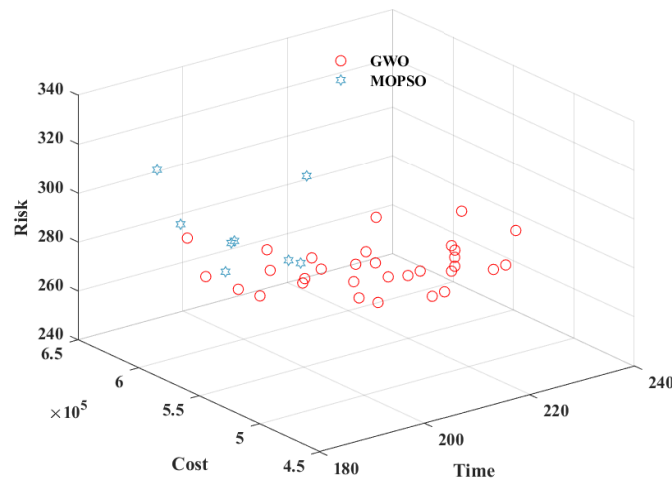


Fig. 3. MOPSO solutions dominated by GWO (own research)

Table 4. Results of MOPSO and GWO (own research)

Algorithm	Variant	MOPSO			GWO			Selected Options
		Time	Cost	Risk	Time	Cost	Risk	
1	Min. time	178	549279.4	330	171	606266.5	345	2,1,2,2,2,3,2,3,4,3,2,1,2
2		179	630593.8	307	172	621540.1	336	1,1,2,1,2,3,2,3,4,2,2,1,2
3		180	567957.5	315	176	633005.4	314	1,1,2,2,2,2,2,3,4,1,2,1,1
4	Min. cost	191	508396.0	333	200	489531.8	329	1,1,2,1,2,2,2,1,3,1,2,1,2
5		181	513268.8	352	188	492774	320	2,1,2,1,2,1,2,1,4,1,2,1,2
6	Min. risk	178	549279.4	330	186	493719.2	340	1,1,1,2,2,2,2,1,4,1,2,1,2
7		197	577482.5	269	198	550538.6	265	1,1,2,1,2,1,1,1,4,1,1,1,1
8		184	647607.7	272	195	633473.3	271	1,1,2,1,2,1,2,3,3,1,1,1,1
9		193	554639.1	277.0	193	542836.8	277	1,1,1,1,2,1,2,1,2,1,1,1,1

Conclusions

Within the scope of this study, two small-scale time-cost-risk problems were examined using the Grey Wolf Optimizer (GWO) algorithm. In the first problem, consisting of 11 activities, different Pareto solutions were obtained with fewer function evaluations compared to the DE algorithm. The hypervolume value achieved by the GWO algorithm for the first problem was calculated as 0.9495. In the second problem, composed of 13 activities, the GWO algorithm presented significantly better results to the decision-maker with the same number of function evaluations as the PSO algorithm. Furthermore, the HV value of the GWO algorithm yielded approximately 10% better results compared to PSO. Upon analysis of the findings, it is evident that the GWO algorithm serves as a favourable alternative for solving time-cost-risk problems.

Bibliography

- Aminbakhsh, S. & Sonmez, R. (2017) Pareto front particle swarm optimizer for discrete time-cost trade-off problem. *Journal of Computing in Civil Engineering*, 31(1).
- Kaveh, A., Rajabi, F. & Mirvalad, S. (2021) Many-objective optimization for construction project scheduling using non-dominated sorting differential evolution algorithm based on reference points. *Scientia Iranica*, 28(6 A), 3112-3128.
- Knowles, J.D. & Corne, D.W. (2000) Approximating the nondominated front using the Pareto Archived Evolution Strategy. *Evolutionary Computation*, 8(2), 149-172.
- Mirjalili, S., Saremi, S., Mirjalili, S.M., & Coelho, L.D.S. (2016) Multi-objective grey wolf optimizer: A novel algorithm for multi-criterion optimization. *Expert Systems with Applications*, 47, 106-119.
- Mohammadi, M.O., Dede, T. & Grzywiński, M. (2022) Solving a stochastic time-cost-quality trade-off problem by meta-heuristic optimization algorithms. *Construction of Optimized Energy Potential*, 11, 41-48.
- Ozcan-Deniz, G., Zhu, Y. & Ceron, V. (2012) Time, cost, and environmental impact analysis on construction operation optimization using genetic algorithms. *Journal of Management in Engineering*, 28(3), 265-272.
- Sharma, K., Soni, A. & Trivedi, M.K. (2023) A particle swarm optimization-based model for quality-safety trade-off optimization under constraint duration and cost of construction project. *Lecture Notes in Mechanical Engineering*, 115-126.
- Sonmez, R. & Bettemir, Ö.H. (2012) A hybrid genetic algorithm for the discrete time-cost trade-off problem. *Expert Systems with Applications*, 39(13), 11428-11434.
- Yilmaz, M. & Dede, T. (2023) Multi-objective time-cost trade-off optimization for the construction scheduling with Rao algorithms. *Structures*, 48, 798-808.
- Zhang, H. & Li, H. (2010) Multi-objective particle swarm optimization for construction time-cost tradeoff problems. *Construction Management and Economics*, 28(1), 75-88.
- Zheng, D.X.M., Ng, S.T. & Kumaraswamy, M.M. (2005) Applying pareto ranking and niche formation to genetic algorithm-based multiobjective time-cost optimization. *Journal of Construction Engineering and Management*, 131(1), 81-91.